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What Makes Some Problems Really Hard: Explorations in the Problem Space of Difficulty

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This paper identifies two sources, one larger, one smaller, of the great difficulty encountered by subjects solving the Chinese Ring Puzzle. With a two hour time allotment, almost none of our college student subjects were able to solve the puzzle unless they were given a demonstration of how to move in the problem space, and even with that help only half of the subjects obtained solutions.

We show that the processing demands imposed by the need to discover how to make moves, rather than other features of the problem search space, are the source of the inordinate difficulty of this problem. Evidence for this view comes from a set of isomorphs that were designed in a manner that "digitized" the moves, which in the original version have analog qualities. These digital isomorphs, which incorporated varying amounts of information about moves, were solvable by almost all subjects, with average solution times of 10 to 25 minutes, depending on isomorph type.

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The digitized problems still required considerable effort for their solution. The difficulty of these problems in digital form, is particularly surprising given the shape of the problem search space. That space is linear: there is no branching. The choice is simply to make a new move or take back the last move. Hence, size of search space (exponential explosion) was not the source of difficulty here. To show this more directly, subjects were started a two different points, 21 and 31 moves from the goal, and there was no difference in difficulty. The linearity of the search space and small number of moves along the minimum solution path (21 or 31) *did not* however prohibit the subjects from making a large number of moves in reaching a solution. The average number of moves ranged from 150 to 450 for different isomorphs. In addition, the subjects' move behavior was often dichotomous, consisting of a very large number of non-progress-making, often error-prone moves, followed by a very rapid, often error-free movement to the goal.

In our examination of the sources of difficulty for this problem, we also studied the transfer of skill between different isomorphs. The investigation of transfer-of-training showed that problem representational features such as move operator compatibility, move difficulty and the presence or absence of move legality cues determined the amount of transfer.

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Abstract

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Introduction

The difficulty of a problem is a major determinant of problem solving behavior and success, and of problem solver frustration. Some problems are so easy that they are usually not considered to be problems at all; others are unsolvable with the resources of time, motivation, and skill that the solver brings to bear. People encounter daily both problems that they cannot solve, and problems that are merely routine applications of automated and effortlessly applied algorithms. Delineating the characteristics that make problems hard or easy is therefore both practically important and theoretically significant. It is the central concern of this paper.

Problem difficulty has usually been attributed to two major sources; the amount of knowledge required to solve the problem (including knowledge of strategies), and the exponential growth of the search space as problem length and complexity increase. The explication of the role of knowledge in problem solving has its modern origins in work on skilled performance in chess that estimated the amount of knowledge required for expertise. (DeGroot, 1966, Chase & Simon, 1973, Simon & Gilmartin, 1973) More recent work has extended those findings to the role of knowledge in problem representation, problem categorization, and strategy selection. (Larkin, McDermott, Simon & Simon, 1980, Chi, Feltovich & Glaser, 1981) While knowledge is of major relevance in understanding problem solving, it is only one of several sources of difficulty.

The second source of difficulty widely recognized is the size of the space that must be searched to find a path to the solution. Newell and Simon (1972) proposed the problem search space as the locus of differences in problem difficulty. Their theory described problem solving as requiring an internal representation of an external task environment. From the possible internal representations of the problem (problem spaces) that might be generated, the subject chooses one or more within which to operate. The size of this search space is an important determinant of problem difficulty because the problem solver has to choose the correct path from start to goal from among the many paths available. Thus such features as length of the minimum solution path, branchiness, and magnitude of possible "garden path errors" have been posited as determinants of problem difficulty. While this quite reasonable view of the problem space as determinant of problem difficulty was dominant for some time, it was challenged by the work of Wason and Johnson-Laird on logic problems, (1972), and Hayes and Simon (1974, 1977) on problem isomorphs of the

Tower of Hanoi problem. Wason and Johnson-Laird showed that the ease of solving problems involving the application of logic principles depended on the familiarity of the domain or the set of materials to which the principles were applied. Domain familiarity does not mean knowledge necessary for solving the problem (such as the relevant logic principle), but rather, familiarity with the materials used (for example, stamps and letters vs. trains and tickets).

Hayes and Simon used sets of problems that were isomorphic (possessed the same structure) but engendered different representations because they were described by differing cover stories. The problems differed in difficulty by ratios (measured by solution time) of two to one. Since the problems were isomorphic, the differences in difficulty could not have derived from the problem search space; this was identical across the various problems. They must lie elsewhere. This, as well as work in other domains, shows that the manner in which problems are represented produces substantial differences in difficulty.

In research aimed at delineating how problem representations affect problem difficulty, Kotovsky, Hayes, and Simon (1985) showed that difficulty was correlated with the size of the memory loads imposed by the move-making processes in different problem isomorphs. They devised difficult problem ismorphs that took 16 times as long to solve as the easiest isomorphs. On all the problems, the subjects spent most of their time learning to make sequences of moves with facility. Virtually all of the difference in time of hard and easy problems occurred during this learning process. Once they could make moves readily, subjects solved hard and easy problems at about the same speed.

What subjects had to learn about move-making in those problems was to plan two moves ahead in order to reach a goal via a subgoal. An analysis of the memory load imposed by planning moves and move-making in the different isomorphs supported the hypothesis that problem difficulty varied with memory demands. Since all of the problems were isomorphs of the Tower of Hanoi, all characteristics of the task--domain-size, branchiness, solution path length, and so on--could be eliminated as causes of the differences in difficulty. Since the task domain was not varied, the studies did *not* show that its characteristics are unimportant, in general, for problem difficulty. They simply demonstrated the importance of a particular variable, memory load, while not precluding the existence of still other sources of difficulty.

The experiments to be reported here investigate a new set of problem characteristics, other than size of task domain, that determine problem difficulty. These new characteristics include the nature of the move itself, and its interaction with other aspects of the task. The starting point for the study is a problem of great difficulty, the Chinese Ring Puzzle.

The Chinese Ring Puzzle and Its Isomorphs

The problems used in the current study are all isomorphs of the Chinese Ring Puzzle which has been described by Afriat (1982). Three of the four isomorphs that are used in this study are depicted in Figure 1.

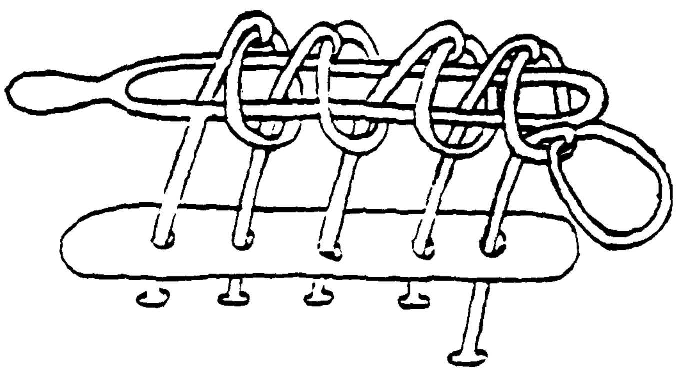
Insert Figure 1 About Here

The first version (Fig. 1a) is the Chinese Ring Puzzle in its traditional form. The task is to remove five rings from a bar on which they are impaled. Unlike such puzzles as the Tower of Hanoi or Missionaries and Cannibals, it is not immediately obvious what constitutes a "move"--that is, what actual manipulations of the physical structure will detach a ring from the bar. It has the analog character of tying shoelaces or a necktie, rather than the digital character of pressing keys on a typewriter. The device is three-dimensional; its parts are loosely joined. There are many ways in which it can be twisted and turned, and in which one part can be slid over another in an effort to make a move. Hence, it poses at least two problems: what sequence of moves will constitute a solution; and how a move is made.

The analog character of the Chinese Ring Puzzle can be seen from Figure 1a. It consists of five steel rings that slide about on a long bifurcated bar which impales all five rings. In addition, five steel cotter pins, one for each ring, are intertwined with the sliding bar and rings in such a way that it is only possible to slide a ring off or on the bar if the ring to its immediate right is on the bar, but all the rings further to the right are off. These two features--that the adjacent ring must be on the bar, and all rings to the right of it off, if a ring is to be free to move--constitute the rules that restrict moves, and thus are part of the move operators. At the usual start of the problem all five rings are on the bar, and the goal is to get them all off. Because our preliminary investigations indicated that this analogical problem was very difficult for our subjects, a number of digital isomorphs were created to enable us to identify the source of the difficulty. Since these problems were isomorphs of

Figure 1: The Chinese Ring Puzzle (a) and two digital isomorphs, No-Info (b) and Lo-Info (c). Moves are made by sliding rings off or onto the horizontal bar (a) or clicking a computer mouse on the box whose ball is to be moved (b & c).

(a)



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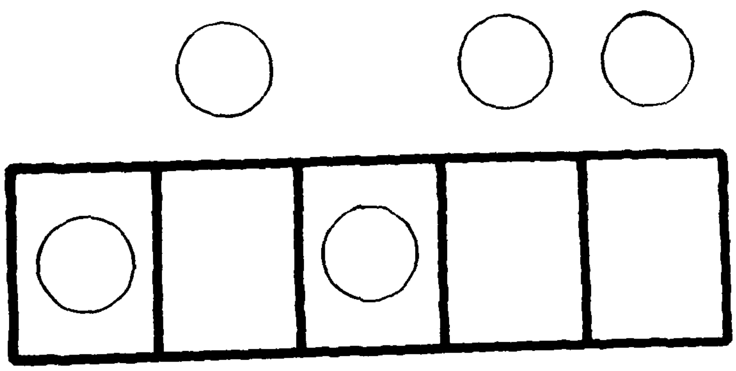

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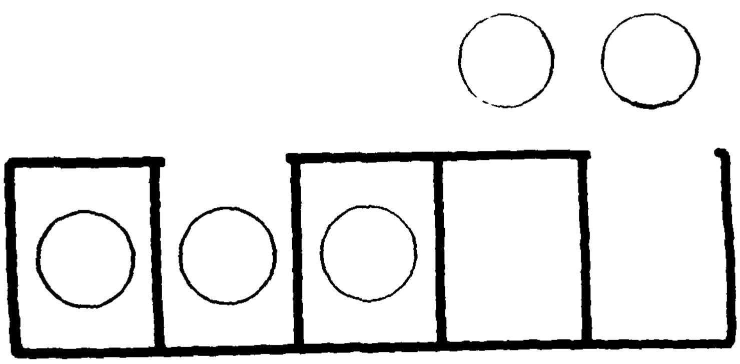

the Chinese Ring Puzzle, they shared its move structure, but embodied different digital move operators. We hypothesized that digitization of the moves would eliminate a major source of problem difficulty. These isomorphs are depicted in the remaining panels of Figure 1. In all of them, the initial position and goal are the same as for the Chinese Ring Puzzle. The digital isomorphs have balls inside boxes that must all be moved out of their respective boxes, instead of rings that must be moved off a bar. The situations are displayed on a CRT, and moves are made by manipulating a mouse.

The second problem isomorph, depicted in Fig. 1b, the no-information (No-Info)[1] problem, consists of a set of five boxes displayed on the screen of a MicroVAX computer. Each box contains a ball that can be moved in or out of it. In the usual starting position all five balls are in their boxes, and the goal is to move all of them out. We call this and the other isomorphs "digital" versions because of the discreteness of the moves, which move a ball in or out of a box, by pointing to it with a mouse and clicking the mouse button.

The third problem isomorph, (Fig. 1c), the low-information problem (Lo-Info), consists of a similar display except that information is provided in the display as to which moves are legal. Specifically, a move is legal if and only if the lid of the corresponding box is open. As the subject makes moves, the lids automatically open and shut according to the rules for legal moves in the Chinese Ring Puzzle, *i.e.*, the ball immediately to the right of the one to be moved must be in its box, and all balls further to the right must be out of their boxes. While this display of the legal moves was potentially valuable to the subject, it did not provide enough information to look and plan ahead, since there was initially no way to tell which boxes would open upon the completion of the current and future moves. In this and the other digital isomorphs, only legal moves are executed. If an illegal move is chosen by a mouse click, the chosen ball does not move.

If the initial situation has all five rings on the bar (all five balls in the boxes), and the goal is to remove them all, then solving the Chinese Ring Puzzle or any of its isomorphs requires a minimum of 21 moves. If the initial situation has only the fifth ring on the bar

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[1]The names of the problems are only moderately descriptive, and should not be interpreted too literally. Thus the "No-Info" problem includes less information in the visual display than the "Lo-Info" problem, but does present some information. The names can be interpreted as an ordering of the amount of display information, rather than a statement of the absolute amount of information displayed.

(only the fifth ball in the box), at least 31 moves are required[2] The reader can verify that, in all but one of the situations that are reachable by legal moves, only two legal moves are available. The exception is that, if only the fifth ring is on the bar (only the fifth ball in the box), there is only one legal move.

Therefore, if the subject has already made some moves, there are at most two moves next available, one of which will return to the position he or she has just left. Thus, the puzzle can be solved blindly, simply by avoiding a repetition of positions! We shall see that this fact is not detected by subjects, and hence does not trivialize the problem. The problem space for the puzzle is shown in Figure 2, which depicts the starting position for the 31-move problem and the 21-move problem, as well as their common goal position, and in addition, depicts the ball (ring) array that corresponds to each state in the search space. It was the 21-move problem that was used in our first exploration of problem difficulty.

Insert Figure 2 About Here

Experiment 1: Digital vs Analog Problems

Subjects

The subjects were 26 students at the Community College of Allegheny County who were paid and/or given class credit for their participation.

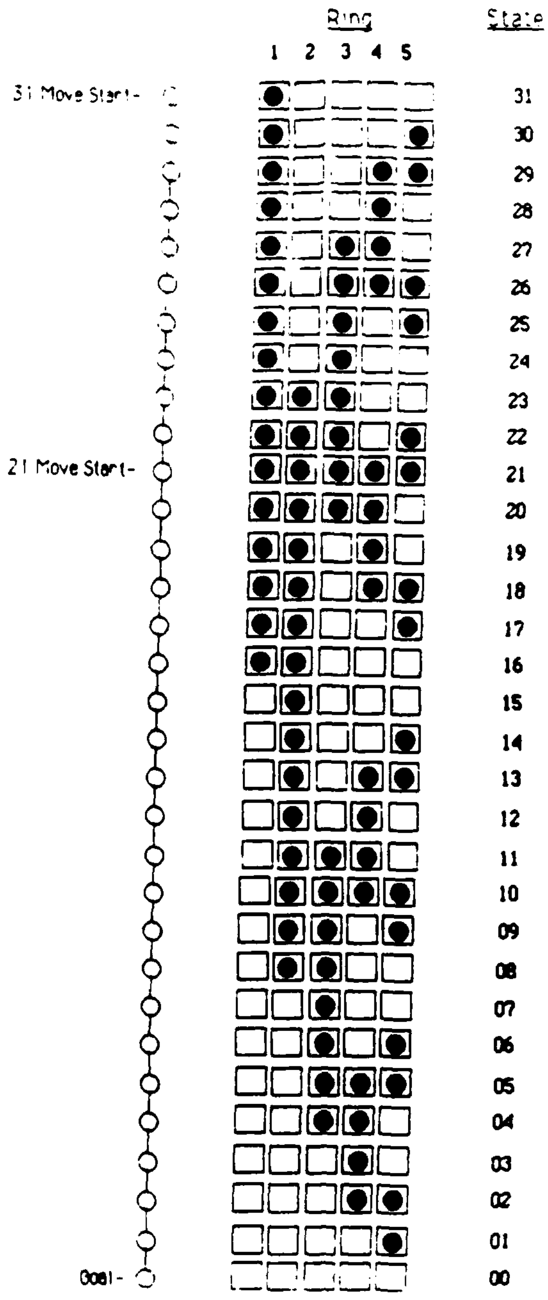
Problems

The problems used in this experiment were the 21-move Chinese, No-Info, and Lo-Info problems depicted in Figure 1, a, b, and c, respectively.

Procedure

[2]In general, for an n-ring problem, there are 2^n possible states in the search space, with a resultant minimum solution path of 2^n-1 moves. There are a number of similarities between this problem and the classic Tower of Hanoi problem. Both problems are infinitely expandible. Another similarity is that in both problems, the most restricted piece (ring or disk) is moved half as frequently as the next most restricted, which is moved half as frequently as the next, and so on. Finally, the minimum solution path length is the same as in the n-disk Tower of Hanoi problem, although the size of the search space is not.

Figure 2: The Chinese Ring Puzzle problem space. The linearity of the space is depicted in the left panel. The right panel shows the corresponding ring/ball configuration. The filled squares denote rings that are on the bar (balls that are in their boxes). The open squares denote rings that are off the bar (balls that are out of their boxes). the most efficient solution of the 31-move problem moves successively from the top to the bottom, passing, en route, the starting point (state 21) for the 21-move problem.



The problems were administered in a transfer paradigm with each subject receiving two problems in sequence, separated by a short rest of about a minute. The method of making moves was demonstrated by showing the subject how the first ball (ring) is moved in and out of its box (off and on the bar). For the computer problems this was done by positioning and clicking the mouse to move a ball out of and then back into its box. For the Chinese Ring Puzzle it was done by sliding the first ring off and then onto its bar.[3] The subject was then timed while solving the puzzle from the 21-move starting position. There was an initial maximum time limit of 60 minutes for the computer problems, and 75 minutes for the Chinese Ring Puzzle. As the difficulty of the Chinese Ring Puzzle became apparent, the time limit was increased to 90 minutes for that problem. Even with the expanded time limit, the problem proved to be virtually impossible to solve. Out of 12 subjects attempting this problem, only 1 solved it, and only one of the others made significant progress toward a solution.[4] While there was an indication that working on the Chinese Ring Puzzle did reduce the time for solving the subsequent Lo-Info problem, the lack of progress on the Ring Puzzle led to our abandonment of the attempt to study it in this unadorned version. Administering a problem with the foreknowledge that people would be unable to solve it seemed both fruitless and unkind; hence the procedure was modified in subsequent experiments.

Results

The primary measure of problem difficulty used in this experiment was solution time. The solution times for the Lo-Info and No-Info problems are presented as solid lines in Figure 3, which depicts the average solution time for the digital isomorphs when presented initially, or after either of the other problems. The solution times for the Chinese Ring Puzzle are not included in the figure because it was solved in the allotted time by only 1 of the 12 subjects who attempted it. (By comparison, only 1 subject of 26 failed to solve a digital isomorph in the allotted time.) The major result of this experiment is to show that the Chinese puzzle is an extremely difficult problem. All but one of our college student subjects

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[3]The first 3 subjects were not given this demonstration on the Chinese Ring Puzzle, but subsequent subjects were. There was no discernible difference in the performance of the two groups.

[4]In the interest of producing solutions, we experimented with a "transfer hint" with four of the subjects, telling them to "use what you learned on this (the first) problem to help solve the second. This manipulation did not yield solutions to the Chinese Ring Puzzle, nor affect the time to solve one of the other isomorphs when they followed the Chinese Ring Puzzle. While the number of subjects is very small, the hint appears not to be a powerful way to elicit transfer.

were not only unable to solve it, but were unable even to approach a solution in an hour and a half. The inability to solve the puzzle was independent of its presentation order: it was virtually impossible to solve whether it was presented first or following the solution of an easier isomorph. Using the times from the problems in initial position as measures of problem difficulty, (and using the maximum allowed time as a solution time for the Chinese Ring Puzzle), the difference among problems was significant ($F(2,21) = 33.72$, $p < .00005$, and each of the other problems differed significantly from the Chinese ($t(11\ d.f.) = 4.98$, $p < .0005$, and $t(14\ d.f.) = 12.17$, $p < .00005$ respectively, for the No-Info--Chinese, and Lo-Info-- pairs.

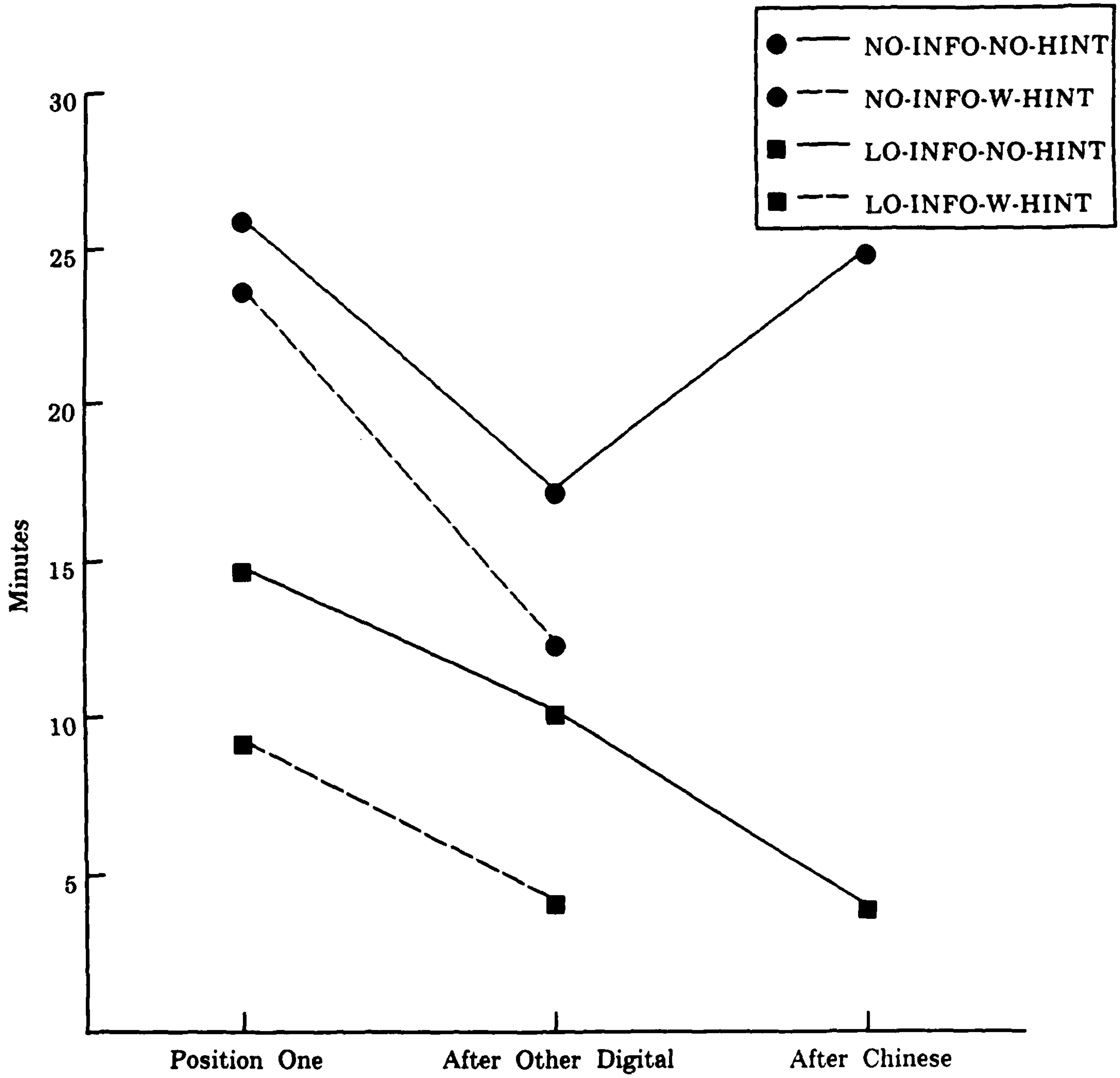
Our experience that the Chinese Ring Puzzle is exceedingly difficult is supported by some early work of Ruger (1910) on a smaller (4 ring and 10 move) version of the puzzle. He found solution times for the problem in initial position, ranging from 2 minutes to 4 hours. The average solution time was 62 minutes and the median 29 minutes for his subjects, who were faculty and graduate students at Columbia University. The finding that it took an average of an hour to solve this much shorter version of the problem, coupled with his fairly select set of subjects, confirms our view of the great difficulty of this problem.

Insert Figure 3 About Here

Both digital isomorphs, by comparison, were easy to solve, taking an average of 25.48 and 14.55 minutes for the No- and Lo-Info problems respectively, in position one. It follows that the difficulty of the Chinese Ring Puzzle lies in features of the problem that distinguish it from the digital problems. Therefore none of the features of the search space account for the difficulty. Similarly, knowledge plays little role in these problems. They are all puzzles which do not draw on prior expertise, knowledge, or analogy. Since digitization of the problem makes it relatively easy, the source of problem difficulty must lie in the move operator of the analog version.

A second finding suggested by this experiment is that the additional information provided by the Lo-Info problem display made the problem easier. The Lo-Info problem was solved in 57% of the time it took to solve the No-Info problem in the first position, and 58% in the second, or transfer, position. This difference in means between the two digital isomorphs,

Figure 3: Solution times: digital isomorphs (Experiments 1 and 2). The average solution time for the No-Info and Lo-Info isomorphs in initial position and in transfer position following another digital isomorph. Solid lines: no hint given; dashed line: strong move hint given.



although sizable, was accompanied by a good deal of subject to subject variation in solution times. It did not reach siginificance in initial position ($F(1,17) = 2.47$, $p < 0.15$), and was only marginal when both initial and transfer position problems are included ($F(1,29) = 3.93$, $p < .06$).

Transfer effects obtained in this experiment are at best small. In the transfer position, the Chinese Ring Puzzle was not solved in 90 minutes, whether following a Lo-Info or No-Info problem. Hence no evidence was obtained for transfer to that problem. With the Chinese Ring Puzzle in initial position, while the number of subjects is small (3 and 4 respectively for the Chinese--No-Info and Chinese--Lo-Info problem pairings), there is some suggestion that Lo-Info subjects might have benefited from prior administration of the Chinese Ring Puzzle. These results, (presented in Figure 3), are a bit surprising given the failure of subjects to solve the Ring Puzzle.

The transfer obtained between the digital problems, the Lo- and No-Info problems, was 32% for the No-Info problem as target, and 31% for the Lo-Info problem in target position. The transfer effect is very marginal ($F(1,29) = 1.89$, $p < .2$), and the approximate equality is surprising in view of the different amounts of information in the two conditions. The subjects presumably could learn more from the Lo-Info problem than from the No-Info problem, and yet, to the extent that any transfer was obtained, the two problems seemed to produce comparable amounts.

The Task Domain

While the major finding of this experiment is the remarkable difficulty of the Chinese Ring Puzzle, both absolutely, and when compared to the digital isomorphs, even the digital isomorphs were not as easy as might be expected from an analysis of the task domain. While it is hard to calculate how long a problem "should" have taken, an estimate can be derived by multiplying the average time to make moves in this problem (3.68 seconds when it is the first problem), by the minimum number of moves required for solution (21). This yields a value of 77 seconds for the minimum expected solution time, or about 13% of the time actually taken by the subjects who solved this problem. Using the move time from the end of the problem, or from the problem in position two, when the subjects are more experienced, and presumably moving at a pace closer to their maximum, yields a value, smaller by an additional factor of four, that is probably closer to the true minimum:

approximately 20 seconds. Hence the representation of the move in the Chinese Ring Puzzle, though the major contributor to problem difficulty, is not the only source. Even the digital isomorphs are surprisingly difficult given the speed with which moves can be made and the linearity of the search space.

Experiment Two: Move Hints

In order to test further our conclusion that the Chinese Ring Puzzle is hard because of the ambiguity of the move-making process, a second experiment was conducted. In this experiment, a move hint was given the subjects before each problem. This move hint, which involved demonstrating a move in the problem while explaining some of the conditions for a move, addresses directly the posited source of problem difficulty. The goal was to ascertain the effect of move operator difficulty on problem difficulty, and to determine factors that control transfer of training between these problems.

Subjects.

The subjects for this experiment were 69 students at the Community College of Allegheny County who were paid for their participation.

Procedure.

The basic experimental paradigm in Experiment 2 was similar to that of the first experiment. The major changes were that subjects were scheduled for a longer experimental session (two and a half hours) in order to give them a maximum amount of time to solve the Chinese Ring Puzzle without exhausting their motivation. Two hours was allowed for the Chinese Ring Puzzle, leaving the remaining half hour for one or the other of the digital problems[5] The second major change was that the subjects were given a strong move hint. There were five subjects per cell in conditions using the Chinese Ring Puzzle, and seven per cell in all other conditions (those using two digital isomorphs).[6]

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[5] In the few cases (3) where subjects had exceeded the planned total time but were still working on the digital isomorph, they were allowed, if willing to continue, to finish the problem, making the effective maximum time, 45 minutes for those isomorphs.

[6] An additional subject was inadvertantly run in one Chinese Puzzle cell, making the total number of subjects 69, 41 in conditions using the Chinese Puzzle together with a computer isomorph, and 28 in the conditions using two computer isomorphs.

Where a hint was given, it was administered in similar fashion in both problems solved by the subject. In this experiment, there were two versions of the hint: the second-ball hint, and the third-ball hint. The second-ball hint consisted of showing the subject how to move the second ball(ring) while telling him/her part of the move restrictions. Specifically, during the demonstration of the move, the subject was told "In order for the second ball(ring) to move out(off) or in(on), the first one must be in(on)". For the third-ball hint, the subject was shown how to move the third ball(ring) while being told "this one can only move if this one (the second one) is in(on)". The subject was allowed to rest for a minute between the administration of the two problems.

The subjects were thus informed of a significant component of the move operator restrictions, while seeing how a move involving more than one ball(ring) was achieved. The second-ball hint has the potential disadvantage of moving the subject in the wrong direction, while the third-ball demonstration has the potential advantage of starting the subject toward the goal. Both hints show the subject a great deal about how to make moves in the various puzzles.

Results: Problem Difficulty.

The results show that the Chinese Ring Puzzle was still very difficult when compared to the digital problems. The solution times for the Ring Puzzle were much longer than for the digital problems even with a move hint. An anova of the problems in initial position yielded an $F(2,63) = 44.96$ $p < .00005$. The average solution times were 79.7 minutes[7] for the Chinese Ring Puzzle, 19.9 minutes for the No-Info isomorph, and 13.0 minutes for the Lo-Info isomorph. The differences were all significant, (Chinese--No-Info $t(43\ d.f.) = 6.58$, $p < .00005$, Chinese--Lo-Info ($t(43\ d.f.) = 7.33$, $p < .00005$, and No-Info--Lo-Info $t(46\ d.f.) = 2.20$, $p < .05$). An analysis of both initial and transfer position problems showed that the effect of problem type was very large ($F(2,126) = 127.36$, $p < .00005$), order (initial vs transfer position) was not significant ($F(1,126) = .22$, $p < .7$), and the type of hint made no difference ($F(1,126) = 1.85$, $p < .2$). There was a marginal interaction between order and problem ($F(2,126) = 2.05$, $p < .15$), with the digital problems a bit easier in transfer position, and the Chinese Ring Puzzle a bit harder. This point is examined in more detail below. A few more than half of the subjects (24 of 41) failed to solve the Chinese Ring

[7]Because of the inclusion of a sizable number of non-solvers whose times were taken as the maximum allowed time, the median solution time was also calculated. It was 81 minutes.

Puzzle in the allotted two hours; while there were very few non-solvers (4 out of the same 41 subjects) on the digital isomorphs in the much shorter time allotted to those problems. However, the Chinese Puzzle, despite its relative difficulty, was solved by a significant proportion of the subjects in this experiment, whichever type of hint they received. This stands in sharp contrast to the previous results that showed almost no subjects solving the puzzle without a hint. The finding that giving subjects the strong move hint did make the Chinese Ring Puzzle solvable, while having a much smaller effect on the digital versions of the problem, (solution times were reduced by only 17%) supports our earlier conclusion about the difficulty of the Ring Puzzle. Removing the uncertainty involved in making moves either by digitizing the move, or by demonstrating it, both make the problem solvable for many of the subjects well within the time limits.[8] Since the effect of both the hint, and digitizing the problem is to define the move better, their efficacy makes inescapable the conclusion that <u>the ill-defined nature of the move is the major source of the great difficulty of the Chinese Ring Puzzle</u>.

<u>Results: Transfer.</u>

While the overall transfer effect was only marginal, a separate examination of the Chinese--digital problem pairs and of the digital--digital problem pairs yields some insights into factors affecting transfer. As pointed out above, the transfer from a digital isomorph to the Chinese Ring Puzzle is not significant ($F(1,39) = 1.22$, $p < .3$). As in Experiment 1, the Chinese Ring Puzzle is no easier in the second position than in the first. As an initial problem, the Chinese Ring Puzzle produced no overall positive transfer effect, but there is a marginal tendency for it to aid the solution of the Lo-Info problem, while having the reverse effect on the No-Info problem. Comparing the two digital isomorphs in initial position and in transfer position following the Chinese Ring Puzzle, there was no significant effect of problem, or order, $F(1,37) = 0.60$, $p < .44$, and $F(1,37) = 0.13$, $p < .72$, but again, as in Experiment 1, there was a tendency for the interaction between problem and order to be significant

[8]All seventeen subjects solving the Chinese Ring Puzzle in Experiment 2 solved it before the 90 minute time limit used in Experiment 1, sixteen of the seventeen in less than 75 minutes. Observation of the behavior of many subjects suggests that this might have been due to motivation: subjects not reaching a solution within 90 minutes might have decreased their efforts, in effect giving up.

$(F(1,37) = 3.05, p < .10)$[9].

One possible reason why the only positive transfer from the Chinese Ring Puzzle in Experiments 1 and 2 was to an easy problem, the Lo-Info problem, is that subjects solving the more difficult target problems might not be able to carry out the transformations that are needed to map one problem onto the other. Their processing resources might not allow them to convert information gleaned from a source problem having quite different move operators into information useable on the transfer problem, with enough working memory left over to solve the problem. While the convergence of the findings of Experiment 1 and Experiment 2 suggest this result and consequent explanation, the marginal significance of each of the two makes the explanation tentative. If this explanation is correct, then a comparison of transfer to hard and easy target problems should yield a more symmetric result when the problems representations are similar and thus allow for an easier mapping of useful information from one problem to another. The ease of mapping would make the difficulty of the target problem less important in the control of transfer, and should result in positive transfer to both target problems. The transfer that was obtained between the two computer isomorphs is in agreement with this prediction.

Figure 3 contains a plot (dashed lines) of the transfer effects of the computer isomorphs paired with each other. The transfer between the No-Info and Lo-Info digital isomorphs was significant $(F(1,48) = 18.03, p < .00025$. The difference in solution times for the two problems was also significant, with the Lo-Info problem taking less time to solve than the No-Info problem $(F(1,48) = 33.89, p < .00005)$. For first problems only, the difference in difficulty is also significant, $F(1,24) = 18.54, p < .00025$. The reduction in solution time through transfer is roughly the same in the two cases, 47.6% from the Lo-Info tc the No-Info isomorph, and 60.5% from the No-Info to the Lo-Info, and the small difference in amount of transfer was not significant $(F(1,48) = 2.03, p < .2)$.

This result is rather surprising, for there is no apriori reason why the Lo-Info problem should produce the same reduction in solution time on the No-Info problem as the No-Info

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[9] If transfer scores are computed (by subtracting the average time in initial position from the time for the same problem in target position and dividing by the time in initial position), the difference between the No-Info and Lo-Info problems is significant, with $F(1,19) = 6.04, p < .025$. This result should be interpreted with caution because the Lo-Info isomorph yielded uncharacteristically long solution times in this portion of the experiment, and the No-Info, uncharacteristically short times.

problem produces on the Lo-Info. It is particularly surprising since the two problems differ considerably in the potential they afford for learning. The Lo-Info problem displays the legal moves at each point in the solution process, and thus should provide more information that is useful on the subsequent No-Info problem than the latter provides for the Lo-Info problem. The reductions in number of moves made in solving each problem are fairly consistent with the reductions in solution times. While the reduction in absolute numbers of moves (from source to target position) is sizable, it is only marginally significant ($F(1,48) = 3.18$, $p < .10$), but the effect of problem is significant ($F(1,48) = 39.07$, $p < .00005$), with the Lo-Info problem requiring fewer moves for solution than the No-Info problem. There was no effect of hint condition. For illegal moves, neither order nor hint condition had an effect, but type of problem did, with many more illegal moves occurring in the No-Info problem than in the Lo-Info problem ($F(1,48) = 66.05$, $p < .00005$).

Discussion.

The transfer results taken as a whole, demonstrate that isomorphism alone is not an adequate predictor of the amount of transfer between two problems. The move operator compatibility, the difficulty of the problems, (and possibly other factors as well), strongly influence the amount of transfer that is obtained when one problem follows another. The major findings about transfer are that:

1. The amount of information provided on an initial problem is not always a good predictor of transfer, even among two isomorphs (No-Info--Lo-Info) that yield a large amount of transfer. The evidence for this is that even up to the point where they solve the initial problem, subjects solving the No-Info problem continue to make many illegal moves, (and backtrack much more than subjects solving the Lo-Info problem). The latter learn almost immediately not to make illegal moves, and learn to avoid backtracking by the time they solve the initial problem.[10] It appears that the prompts given to the Lo-Info subjects, the depiction of the legal moves, were not useful (and were possibly even counterproductive) for transfer. This contrasts with their marked effect in reducing problem difficulty. The prompts may have simply been followed without teaching anything about the conditions for legal moves, in which case the subjects did not gain knowledge they could apply to the second problem, where such prompts were not present. Having been helped through the first problem, subjects, (like students in many situations), might have been at a loss when faced with an unprompted problem. Another possibility is that the Lo-Info problem, being easier, is more amenable as a target problem to receiving the benefits of transfer. The tendency of the Chinese Ring Puzzle to yield positive

[10] *A more detailed analysis of illegal moves and bactracking is presented later in the discussion.*

transfer to the Lo-Info problem supports this latter interpretation. Since more detailed subject by subject analysis of the transfer of subjects who exhibit varying amounts of learning on the first problem might clarify the situation, it will be reported after an analysis of the moves is presented.

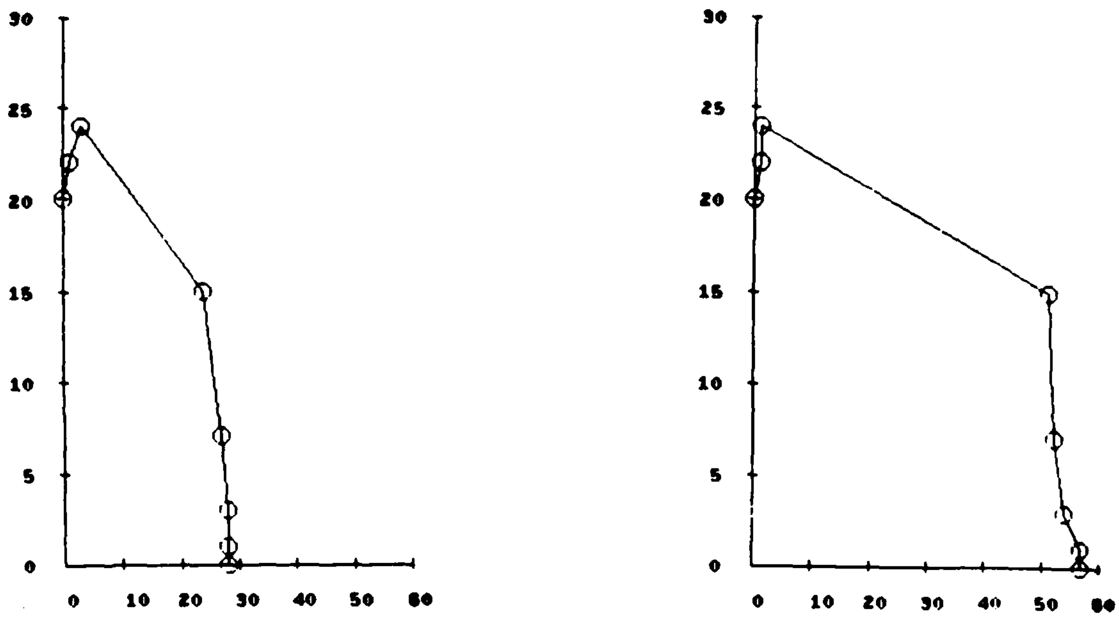
2. The results with the Chinese Ring Puzzle show that when the target problem is difficult, the amount of transfer may be minimal or even negative. One possible explanation is that when problems are dissimilar, the information obtained from the initial problem must be transformed in various ways to be applicable to the transfer problem. The resources required for this transformation might compete for processing resources required for the solution of the transfer problem, thus producing little or no, or even negative, transfer.

3. The general finding that isomorphism is not of itself enough to guarantee a significant amount of positive transfer is in accord with the findings of Hayes and Simon (1977) and Kotovsky, Hayes, and Simon (1985). Factors such as external representation, move operator compatibility, initial problem difficulty, and transfer problem difficulty all seem to be major determinants of transfer.

To cast further light on these transfer effects, the pattern and times of moves of individual subjects on the Chinese Ring Puzzle were analyzed. We recorded the moves subjects made, and in particular, the time at which each ring was removed for the first and last time. The first set of data presented show the times of these major moves. (Since ring five was often left off once removed, we provide only the 'first off' time.) The subject was thus timed at up to nine points in the problem space that correspond to the completion of nine subgoals (ring removals) that are encountered along the way from start to finish. This analysis of the problem space allows us to develop a naive model of expected move times based on how far from the initial position (in number of moves) each subgoal is. Under the assumption of equal times per move, the number of moves from the beginning of the problem provides a prediction of the time to reach the new subgoal. This assumption would yield a linear function relating problem space position to move number or time. The results for two typical subjects, presented in Figure 4, show an accelerating progress (usually containing an inflection point) rather than a linear one. This acceleration, which shows a good deal of variability, but usually starts when the subjects reach position 15, indicates that subjects start behaving differently after getting part way through the problem. This more efficacious movement results in their traversing rapidly the greatest portion of the search space, after spending a relatively long time without making much progress toward the goal.

Insert Figure 4 About Here

Figure 4: Subgoal completions: Chinese Ring Puzzle. The times at which subjects reached particular positions in the problem space are plotted on the abscissa against the corresponding position in the search space on the ordinate. The acceleration in rate of closing on the goal state indicates that most of the problem solving time was spent without making much progress toward a solution.

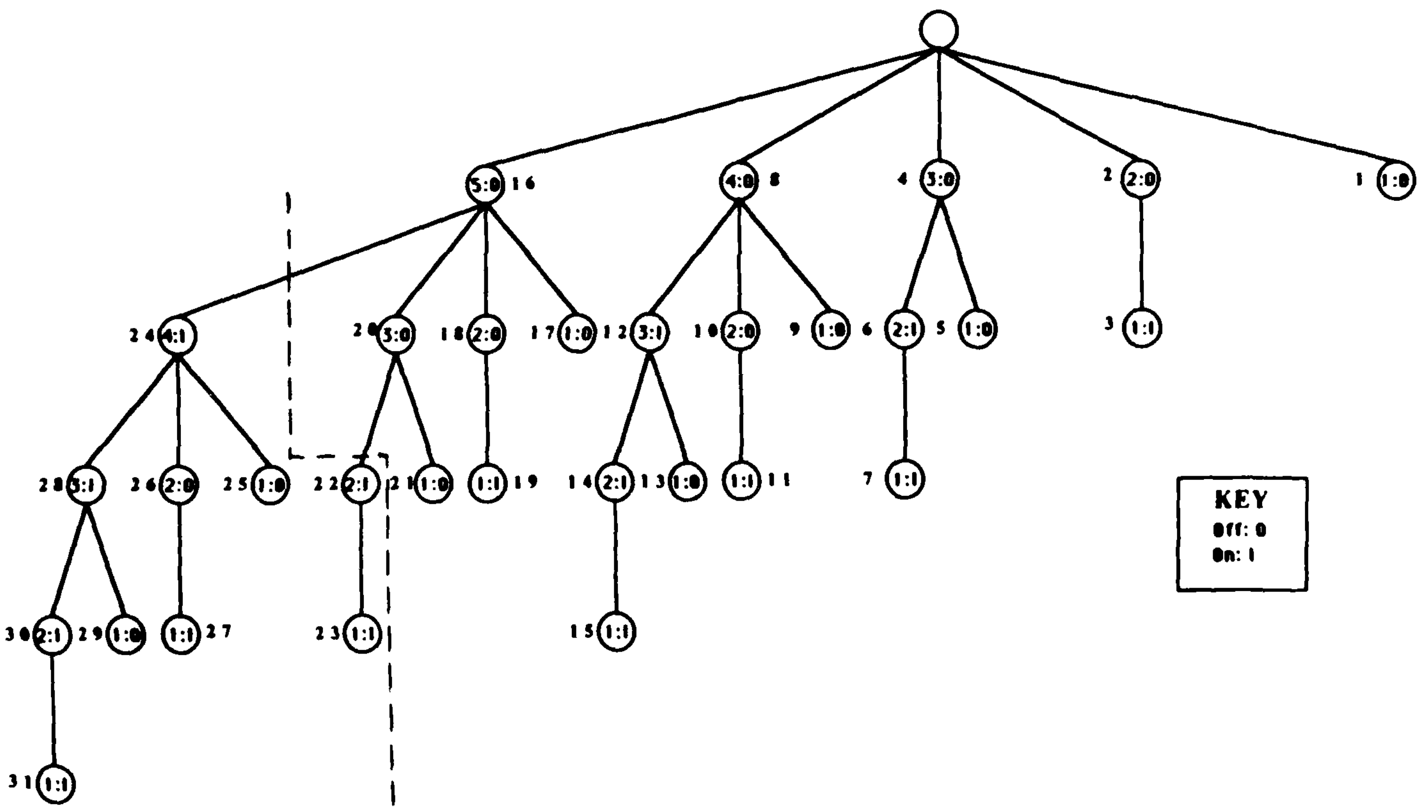


As can be seen in Figure 4 the number of moves to the goal does not predict the subjects' times closely. the steepness of the curve at the end of the problem is due to the fact that once subjects remove the fourth ring after having removed the fifth, they solve the problem rapidly, removing the remaining rings in less than a minute. This result is not due to the effect produced by the failure of almost half the subjects to solve the Chinese Ring Puzzle, for if the analysis is restricted to only those subjects who solved the problem, the result is the same; once the fifth and then the fourth ring is removed, the problem is essentially solved: subjects only require an additional minute or so to remove the rest of the rings. In Figure 5, we depict the goal heirarchy for the problem. The number of nodes below a particular node, measures the number of subgoals that must be reached to complete any goal move. Examination of the goal heirarchy in the figure reveals that the number of subgoals is only minimally predictive of the time to traverse different sections of the problem search space. While the acceleration at the end of the problem generally occurs when the 5th or 4th ball is taken off, the remaining depth of the goal heirarchy is substantially equal to that in earlier parts of the problem, and thus cannot account for the acceleration that is found for most subjects. Similarly, a move-by-move analysis of the depth of the goal heirarchy predicts that the longest move times should occur before moves at position 31 and 23 (in the 31 move problem), and move 15 in all problems. Inspection of the subjects' data shows that this was not the case. The inability of the goal heirarchy to predict move latencies shows that subjects did not solve these problems by planning long sequences of moves.

Insert Figure 5 About Here

The inflection point in progress toward the goal was explored further by collecting detailed move records for each subject on the computer isomorphs. These move records were collected for the Lo-Info/No-Info pairings of Experiment Two. and in two subsequent experiments as well. The results of these latter experiments will be presented before the details of moves are analysed.

Figure 5: Goal heirarchy: Chinese Ring Puzzle. The goal heirarchy for the 21-Move Puzzle and, with the inclusion of the goal structure to the left of the dotted line, the 31 Move Puzzle. The circles represent problem goals and subgoals. The number to the left of each circle is the state in the problem space that is attained by that move. The number in the circle is the number of the ball that is moved, and the I/O represent the movement of a ball(ring) in(on) or out(off) respectively. All subgoals below each goal node must be completed before that move can be executed. Thus for example, near the end of the problem, after ball 3 is removed (state 4), the removal of ball 2 is the next goal (state 2) but ball 1 must be put into its box (state 3) before ball 2 can be removed.



Experiment Three: Representation and Planning

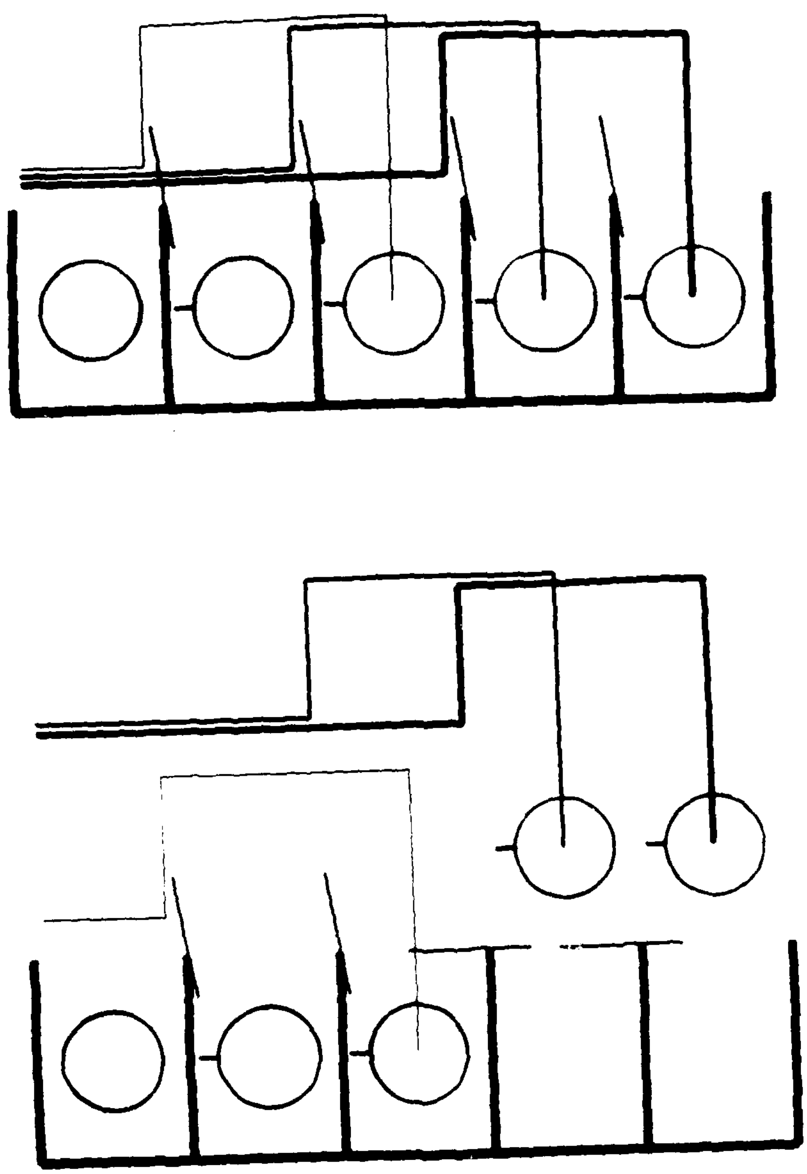
The results of Experiments One and Two demonstrate the advantage of digitizing the Chinese Ring Puzzle, and the further advantage of providing solvers with information pointing them to legal moves (the Lo-Info condition). Within the digital isomorphs, by designating which moves are legal at every step, the problem solving time can be reduced by 50% or more, and the number of moves is reduced even more. The transfer data on the other hand, are mixed; while digitizing does aid transfer (certainly eliminating the negative transfer that is found with the extremely hard Chinese Ring Puzzle), the Lo-Info problem, displaying move information, did not facilitate transfer. In discussing this finding earlier, we suggested that providing information about legal moves might have two effects: learning more about the problem should aid the solution of the transfer problem, but providing a "crutch" might aid problem solving without inducing the learning that could be transferred. In target position however, the information provided by the Lo-Info problem did enhance transfer.

On the basis of our initial findings we designed a new isomorph incorporating move legality information that could promote the development of a mental model of the problem transferable to subsequent problems. This isomorph, depicted in Figure 6, is designated the All-Information (All-Info) problem because the computer displays a set of gates and blockers that determine the move restrictions. Thus it is logically possible for the solver to construct a mental model of the move operators and the problem space by examining and understanding the restrictions, depicted on the screen, whereby one ball blocks and unblocks another. This representation was necessarily complex, and to insure that subjects were able to access it, its major features were explained to them just before they began work on the problem. It is in principle possible for the subject to look at the initial display and plan a sequence of moves to reach a solution. But the complexity of the display makes this possibility problematic, and thus whether it is real must be resolved empirically.

Insert Figure 6 About Here

This isomorph models the move restrictions by means of attachments to the balls that block and unblock other balls' movement. In addition, levers that are moved by the balls open and close other boxes. The operation of the attachments is depicted in Figure 6 which shows (a) the position 21 start, and (b) position 19 that results from two sequential

Figure 6: The All-Info Problem. The short attachments sticking out of the left side of each ball flip the box lids open or closed as the balls move in or out of their boxes. The long wire-like structures attached to each ball except the leftmost constrain the movement of balls to the left of the ball the wires are attached to.



legal moves toward the goal. The box lids are opened and closed by the small portrubances attached to the left side of each ball, and the balls are blocked by the long attachments to the tops of the balls. As in our other experiments, the experimental paradigm was a transfer paradigm that involved the sequential presentation of two problems.

Subjects

The subjects were 42 students from the Community College of Allegheny County who were given class credit for their participation.

Procedure

The subjects were presented with two digital isomorphs, an All-Info and either a Lo- or No-Info problem, presented on a MicroVAX computer. In the first condition of this experiment, 14 subjects were presented with an All-Info and then a Lo-Info problem. The All-Info isomorph did not make the problem more "understandable": it did not enable people to solve the problem more quickly than the No-Info problem with similar hint conditions, nor did it eliminate illegal moves, which is what the display was designed to do.[11] The mean number of illegal moves with the All-Info problem was 206 per subject, suggesting that the display of the move restrictions was not effective. As a result of this finding, the experiment was changed to give subjects in the All-Info condition an explanation of the display. The subjects were told what the effects of the blocking features of the display were. Thus, while pointing at the lines attached to a ball, (Figure 6) the experimenter said "This allows this ball to move but blocks these from moving.", and, while pointing to the levers on the boxes said "The balls move these as they move in and out of the boxes, opening and closing the adjacant box to movement of its ball." This explanation was effective in enabling subjects to avoid making illegal moves; the number was reduced to a mean of 46 per subject. The All-Info problem was used with this explanation in subsequent experiments, and it will subsequently be referred to simply as the "All-Info Problem". The All-Info problem was paired with the No- and Lo-Info problems in the same kind of two-problem transfer paradigm that has been used throughout this work.

Results

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[11]The All-Info problem was the first isomorph that attempted to display the move restrictions in such a way that the subject could visualize the effects of making moves upon the legality of succeeding moves, and thus plan a move sequence.

In the initial position, the All-Info problem took the longest to solve, followed closely by the No-Info and then the much faster Lo-Info problem. The mean solution times for the three problems (all using the 2nd ball hint), were: 20.37, 18.92, and 12.29 minutes, respectively. An Anova for the three isomorphs yields an $F(1,62) = 6.31$, $p < .005$. An analysis of the differences between the various pairs of problems shows the No--Lo and All--Lo differences to be significant ($t(41$ d.f.$) = 2.75$, $p < .01$, and $t(42$ d.f.$) = 3.56$, $p < .0025$) respectively. The All--No difference was not significant, $t(41$ d.f.$) = .57$, $p < .6$. These results are displayed in Table 1. Not all of the information provided by the All-Info representation was usable, even with the explanation of the display. The paucity of illegal moves shows that the move legality information was usable, but that the "mental model" information was not as useful as the more limited information of the Lo-Info isomorph. Two explanations are possible: (1) the additional information somehow overloaded the processing system, or (2) the additional information was useful, but slowed solution time because using it required a significant amount of time. If the latter is the case, we would expect the time per move to be much higher in the All-Info problem than in the Lo-Info problem. Table 1 gives the time per move for the three problems.

Insert Table 1 About Here

As can be seen in the Table, the time per move is much higher for the All-Info problem than for either of the other digital isomorphs in both the initial and transfer positions. Both positions yielded highly significant differences. For the initial position we have $F(2,56) = 17.79$, $p < .00005$, revealing a much slower move time for the All-Info isomorph. The transfer (decrease in time per move from initial position to transfer position) was also significant ($F(1,56) = 44.98$, $p < .00005$), and the interactions showed that the amount of transfer depended on both the initial problem ($F(2,56) = 29.67$, $p < .00005$), and the transfer problem, ($F(2,56) = 11.97$, $p < .00005$), as well as the interaction between the initial and transfer problems ($F(4,56) = 2.6$, $p < .05$).

Our pilot experment with the All-Info problem without explanation provides further evidence that using the information in the All-Info display was time consuming. If the external representation was not useable without an explanation (as the illegal move data suggested), and if using the display information slows subjects in the All-Info (with explanation) problem,

Table 1: Solution Times, Number of Moves, and Time per Move measures for the All-Info, Lo-Info, and No-Info digital isomorphs in initial and transfer position.

	Position One	After No-Info	After Lo-Info	After All-Info
Solution Time (min.):				
No-Info	18.92	6.60	12.36	12.82
Lo-Info	12.30	5.13	2.96	3.82
All-Info	20.37	13.69	11.67	5.76
Moves:				
No-Info	420.6	205.3	331.6	297.9
Lo-Info	149.7	121.0	53.8	78.6
All-Info	180.8	156.3	107.1	66.4
Time per Move (sec.):				
No-Info	3.03	2.00	2.45	2.83
Lo-Info	5.34	2.56	3.91	3.53
All-Info	8.03	5.71	6.26	5.58

then moves should be made faster in the condition without explanation than that with explanation. Examination of those data shows a 34% difference in speed of moves in the expected direction.

Because of the differences in move times, we will also measure problem difficulty by the number of moves to solution (Table 1). In initial position the problems differed significantly by this measure ($F(1,62) = 11.7$, $p < .00005$). The No--Lo difference was significant ($t(41$ d.f.$) = 4.65$, $p < .00005$), the No--All difference was significant ($t(41$ d.f.$) = 4.02$, $p < .00025$), and the Lo--All difference was not significant ($t(42$ d.f.$) = .96$, $p < .4$). The All-Info problem was thus similar to the Lo-Info problem by this measure and easier than the No-Info problem.

When the amount of transfer to or from the three isomorphs is examined, the problems differed significantly as recipients of transfer ($F(2,56) = 4.31$, $p < .025$, with the Lo-Info benefiting the most as a recipient, while not serving as a particularly effective source. The source differences did not reach significance ($F(2,56) = .34$ $p < .75$). The transfer effects are presented in Table 2. Examination of that figure reveals that for each problem, the most effective source of transfer was, not unexpectedly, itself. In fact, for the No-Info and All-Info problems, such self transfer was the only effective source.

We conclude that the mental model provided by the All-Info problem did not provide particularly useful transfer information. While it appears from the time-per-move data that subjects were trying to use the display, its effectiveness for transfer was no greater than that of the No-Info isomorph.

Insert Table 2 About Here

In summary, the All-Info problem representation is somewhat useful for problem solving, although not as useful as the unmotivated marking of the legal moves in the Lo-Info condition. A similar conclusion is reached regarding usefulness for transfer. Taking these findings together with the somewhat similar findings of Experiment 2, it appears that the significant variable determining problem difficulty and transfer is the nature of the move. The digital or analog nature of the move is the major determinant of problem difficulty and to a

Table 2: The amount of transfer (percentage reduction in solution time) obtained between the three digital isomorphs.

	Transfer Problem Type			Mean
	No-Info	Lo-Info	All-Info	
Source Problem				
No-Info	64.8	58.1	32.9	51.9
Lo-Info	34.4	75.7	42.8	52.1
All-Info	32.0	68.7	71.8	58.2
Mean	43.7	67.9	50.2	

lesser extent, transfer. The other differences we have introduced (those within the digital problems), have substantial but much smaller effects. The differences between the Chinese Puzzle and the digital isomorphs dwarf the differences among the digital isomorphs.

Experiment Four: Problem Space and Difficulty

In this experiment, we investigate the effect of problem space size on problem difficulty by having the subjects solve a digital isomorph of the Chinese Ring Puzzle from two different starting positions, 21 and 31 moves from the goal. The two starting positions, indicated in Figure 2, represent two "natural" places from which to start the problem: (1) the point with the longest minimum solution path (31 moves), and (2) the point where all the balls are in their boxes (21 moves). The goal for both problems was the same, the removal of all the balls.

Subjects

The subjects were 42 students from Community College of Allegheny County who received class credit for their participation. Of these, 14 were new subjects who were run under conditions identical to those of Experiment Two, except for the changed starting position, and 28 were subjects from that earlier experiment whose data were used as the 21-move condition control.

Procedure

The same procedure was used as in the previous experiments. Each subject was given in sequence two problems, presented on the MicroVAX computer. The two problems were the Lo-Info and No-Info isomorphs of the Chinese Ring Puzzle, with hint 2, described in Experiment Two. Half of the subjects received the problems in the order Lo-Info/No-Info, and half in the order No-Info/Lo-Info. Both of the problems a subject was given were of the same length, either 21 moves or 31 moves. The basic measure of problem difficulty is the solution time. The results are depicted in Figure 7.

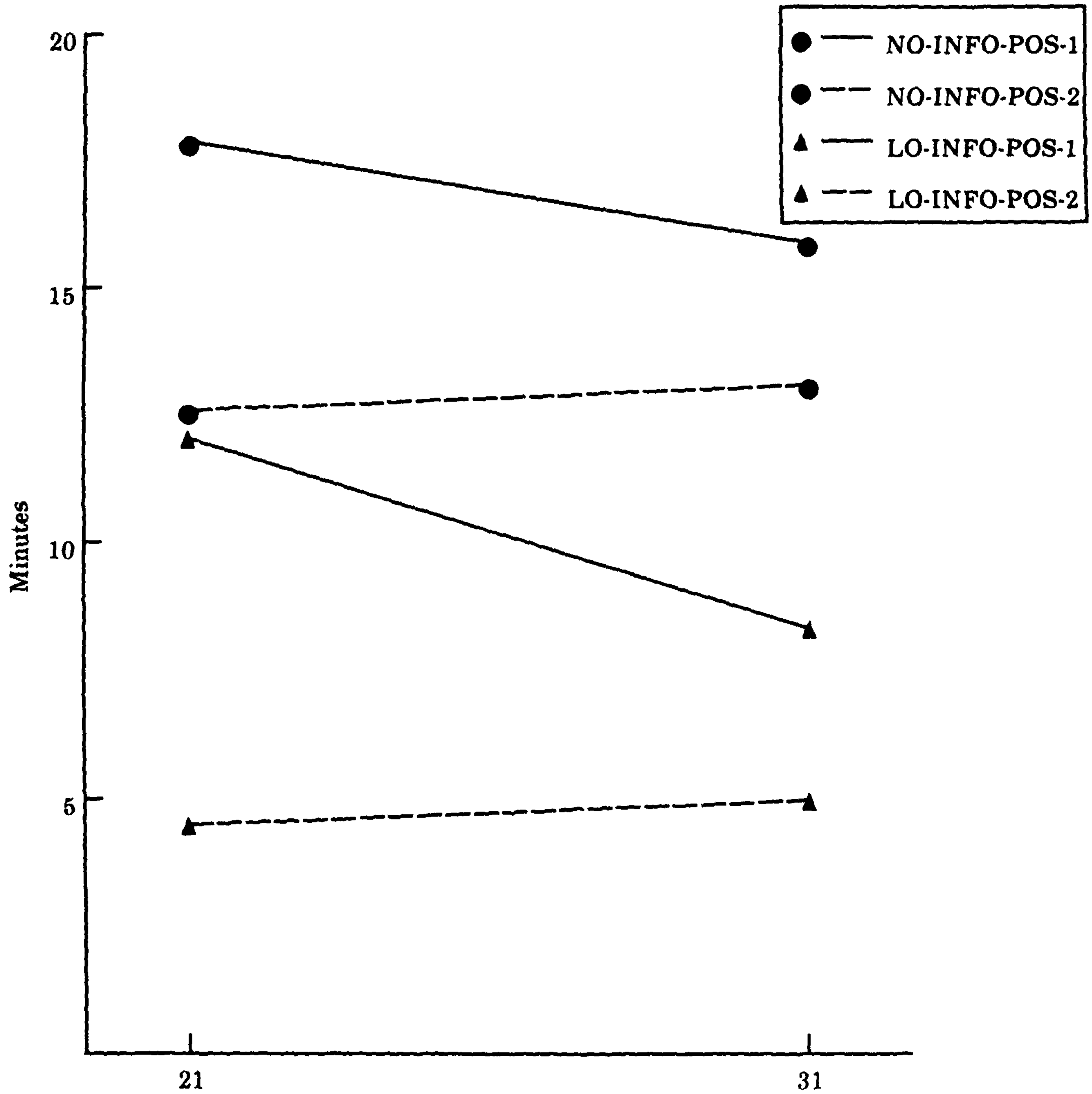
There was no significant difference between the 21- and 31-move problems in solution time ($F(1,53) = 2.12$, $p < .2$ for initial problems, and $F(1.62) = .95$, $p < .50$ for initial and transfer problems). There was a small tendency for the 21-move problem to be a bit harder in position 1, a result opposite to that which would be expected from the path length. An analysis of movemaking again shows no significant difference due to starting position. In initial position, the No-Info problem was solved in fewer overall moves than the Lo-Info problem ($F(1,53) = 15.27$, $p < .0005$, but the starting position produced no significant effect on the number of moves, $F(1,53) = .88$, $p < .5$).

Insert Figure 7 About Here

The surprising conclusion from these data is that the length of the minimum solution path does not contribute to problem difficulty. Our reluctance to accept this result led us to investigate a possibly spurious reason for the finding. People starting from the 21-move position might head in either the right or wrong direction; that is, either toward or away from the goal. Either direction will eventually lead to a solution. Those who head in the right direction simply have a minimum of 21 moves, while those who initially head in the wrong direction need a total of 10 plus 31 or 41 moves, (the 10 moves from the 21-move starting position to the 31-move start, and then the 31 moves to the goal). If subjects split about evenly in choosing the two directions, then they would on average, have the same path length as the 31 move subjects (21 + 41)/2. The move records of the subjects in the 21-move condition reveals that only 4 of 14 subjects did use this long path in position one, while 12 of 14 did in position two. This total of 16 choices of the long path (out of 28 subjects) is close enough to 50% to suggest that the average of the 21 and 42 move paths could explain the similarity in solution times for the 21-move and 31-move problems.

However, this is not the case. Subjects solving the problem in initial position almost always chose the shorter solution path, while in the transfer position the shorter path was the choice of only 2 of the 14 subjects ($\chi^2 = 7.145$, $p < 0.01$). If path choice is an important determinant of problem length, then the 21-move problem should have been relatively easier in initial position (where the short path was chosen), and relatively harder in target position (where the long path was chosen). The results were the opposite. Figure 7 shows that it is in position one that the 21-move problem tends to be a bit harder, while in position two the 21 and 31-move problems are about equal in difficulty. Calculating more precisely the minimum length of path by taking into account the actual numbers of subjects who solved the 21-move problem via the long route, we get an average for the initial problem (based on 10 subjects at 21 moves and 4 at 41) of 26.7 moves, while for the

Figure 7: The effect of starting position on solution time. 31 Move and 21 Move problem solution times for initial (position 1) and transfer (position 2) problem positions.



target problem, the estimate (based on 12 subjects at 41 moves and 2 at 21) is 38 moves. Thus the 21-move problem should be harder than the 31-move problem in position two, and easier in position one. In fact, there was no significant difference, with a small tendency in the opposite direction.

The conclusion we reach is that minimum length of solution path is not a cause of problem difficulty in problems of this sort. This conclusion is in accord with previous work showing large differences in problem difficulty in problems that are isomorphic. Still, a more detailed analysis is needed if the effect of the problem search space on problem difficulty is to be understood. Many subjects work very hard, often making hundreds of moves in solving these problems, when a very simple strategy could guarantee a solution in 21 or at most, 42 moves. There are interesting patterns of moves in the subjects' behavior that can help explain this result. These will be presented after an analysis of the size of the problem space.

The Size of the Problem Space

We have seen that the problem space for the Chinese Ring Puzzle and its isomorphs is linear. In each state, there are at most two moves--one leading toward the goal, the other away from it. A subject who does not solve the problem in the minimum number of moves must, from time to time, be changing direction, returning to a position just left and perhaps continuing for some moves in the new direction before reversing again.

It will be instructive to model this behavior as a random walk, on the assumption that at each step the subject has a probability, p, of continuing in the current direction, and a probability, $q = 1 - p$, of reversing direction. The number of moves to solution will then be a function of p, and we can ask what values of p would correspond to the path lengths of our subjects.

Our colleague, John R. Anderson (personal communication), has shown that the expected number of moves required to traverse an entire chain of n steps from the beginning is $M(n) = (n+1)(n-(n-1)p)/p$. Hence $M(30) = 31(30 - 29p)/p$. For the 21-move puzzle, where the starting position is not at the end of the chain, $M(21) = M(31) - M(10)$; for solving this puzzle is equivalent to solving the 31-move puzzle after state 21 has already been reached. But reaching state 21 from starting state 31 is equivalent to solving a 10-move puzzle. By examining the formula, it can easily be seen that, for any value of p, the expected number of moves to solve the 31-move puzzle through choosing moves randomly will only be about ten per cent greater than the expected number for the 21-move puzzle, and that this ratio is nearly independent of p. The minimum solution path is of course, about fifty per cent longer (31 vs 21 moves).

If the probability (q) that the subject will reverse direction on any given move is *1/5*, (the observed value for the subjects in the No-Info condition) then the expected number of moves to solution will be *242*, which agrees closely with the observed average number of legal moves in this condition. From this agreement of numbers, we should not jump to the conclusion that the subjects were actually choosing their direction at random. In the next section, we will examine the patterns of moves made by subjects, and will see that these patterns are far from random, and can be given meaningful interpretations.[12]

Patterns of Moves in the Search Space

Starting with Experiment Two, detailed records were kept of the subjects' moves. The computer recorded the time and identity of each legal and illegal move. Moves (without their times) were plotted as a function of the position in the problem space in order to discover repeated patterns in the moves subjects make in solving the problems. Figure 8 a, b, and c, shows the moves made by typical subjects in each of the three conditions, All-Info, Lo-Info, and No-Info. The number of moves is on the abscissa, and the problem space position (number of moves from the goal) on the ordinate. Figure 8 (d) presents one subject's moves on both the Chinese and the preceding Lo-Info problem. The graphs are similar, both exhibiting the acceleration to the goal that commonly occurs late in the problem. Comparison of the move patterns presented here with the Chinese Ring Puzzle moves of Figure 4 also reveals the basic similarity in move patterns (despite the very large differences in solution times).

Insert Figure 8 About Here

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[12]We also explored the size of the problem space by constructing a random walk computer model that implemented the mathematical model in a manner yielding not only the expected number of moves to a solution, but also information about the patterns of moves the model made in solving the problems. The move patterns obtained with this model were quite different from the move patterns exhibited by the subjects.

Figure 8: Examples of typical move records: Digital Isomorphs. The number of moves is printed on the abscissa, and the problem position (with the goal state equal to 0, and the start at either move number 21 or 31), on the ordinate. (a) All-Info. (b) Lo-Info. (c) No-Info. (d) An example of a detailed move record from the Chinese Ring Puzzle together with the initial problem for that subject, the Lo-Info isomorph.

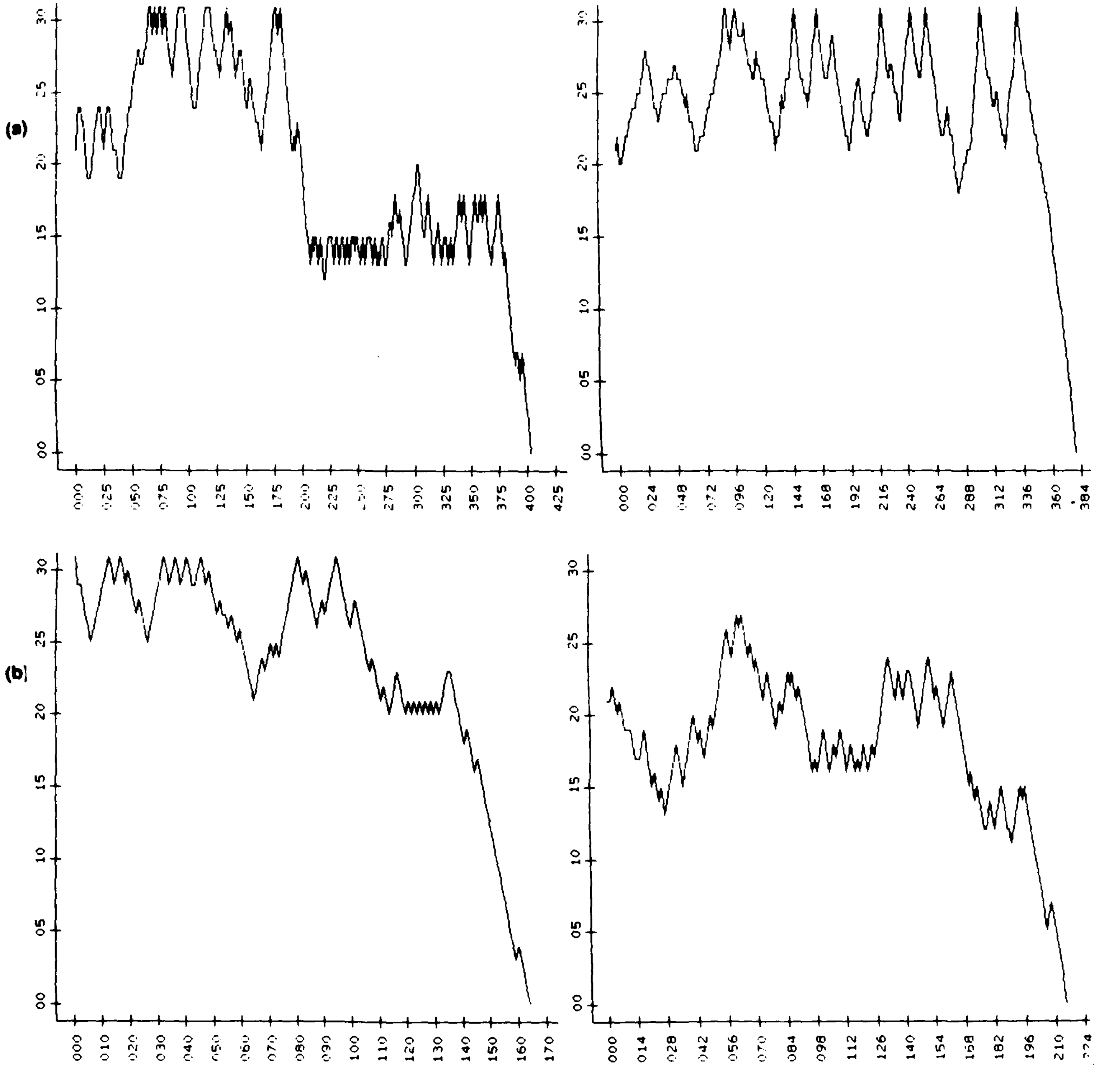
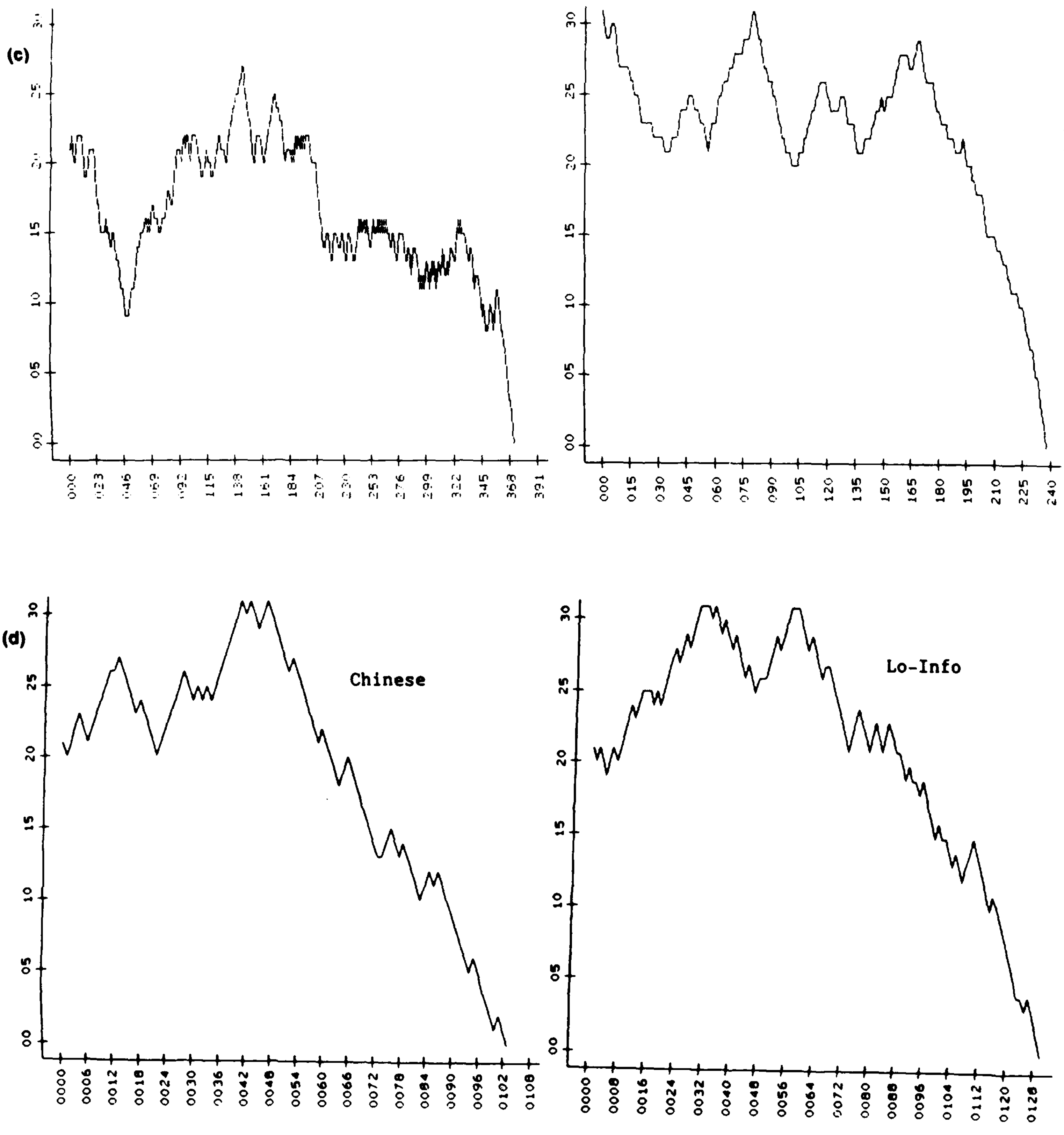


Figure 8 continued



An interesting feature of these move records is that many of the subjects appear to oscillate when they are at certain positions. or within a narrow band of positions. Figure 9 presents some of the details of this behavior, averaged over all three types of digital isomorphs. The places where multiple and quickly repeated reversals of direction occurred (where in effect, the subjects' behavior seemed to indicate that they were unable to penetrate a barrier) are plotted there as a function of problem search space position and direction of travel--toward or away from the goal. Many of them, having started off in the wrong direction, spend a large fraction of their time between positions 21 and 31, often oscillating between 31 and the positions immediately adjacent to it. Once they have reached position 15, they seldom regress to the region above 21, but the region just below 15 is often a zone of oscillation. In only a few cases does a subject regress after reaching position 10. Almost all subjects appear to exhibit a final sprint to the goal, often from a rather distant starting position.

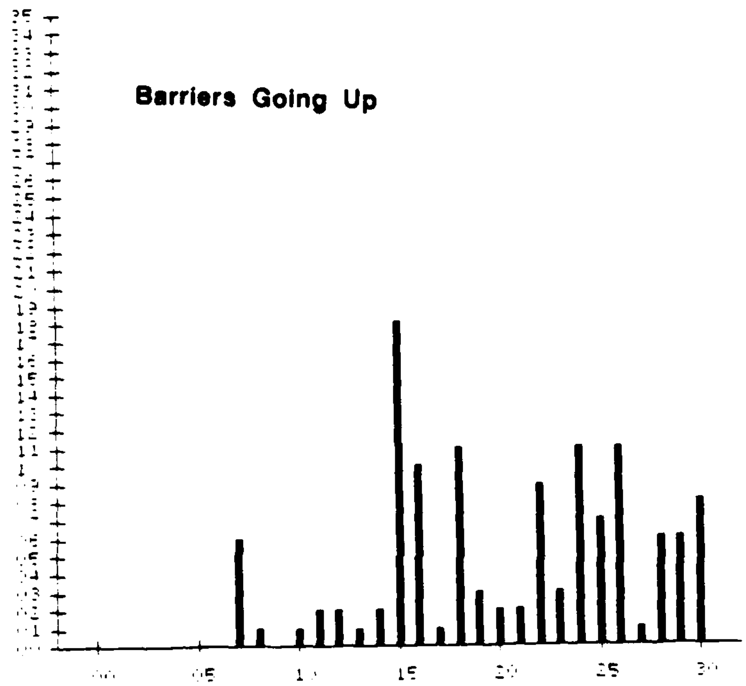
Insert Figure 9 About Here

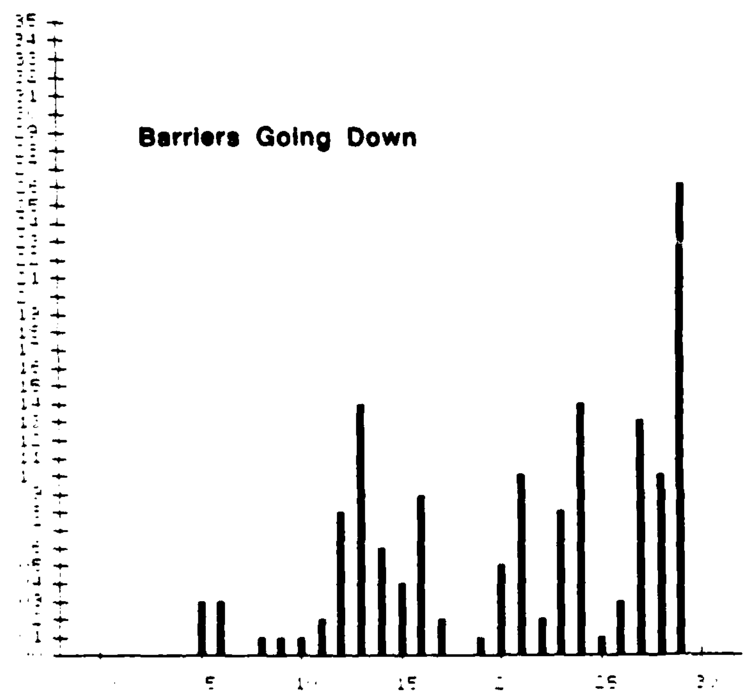
Much light is cast on this behavior if we assume that subjects pay attention to the number of balls that remain in the boxes, and use the reduction of this number as a test of progress. The number of balls in boxes gradually rises to all five in 21. Hence, most of the moves on the path from 31 to 21 are counter-intuitive: seven of them increase the number of balls that remain in the boxes while only three decrease that number.

On the other hand, four of the five moves from position 20 to 15 decrease the number of balls in the boxes, with only one remaining in a box in position 15. Moreover, in position 15, the fifth ball has been removed from its box for the first time. However, the moves from 14 to 10, where four balls are in boxes, are again counter-intuitive, four out of five increasing the number. From position 10 to 7, three consecutive moves reduce the number to one again, and both balls 5 and 4 are out of their boxes for the first time. The number increases momentarily to three in position 5, then four of the remaining five moves decrease it, until it reaches zero at the solution.

We would expect subjects to reverse direction more often when their next move adds a ball to a box (an apparently regressive move) than when their next move removes a ball

Figure 9: Barriers encountered as the subject moved toward ("down") or away ("up") from the goal. (a) & (b) Frequency of barriers. A position was counted as exhibiting barrier- like qualities if a subject reversed his or her direction of travel at that point at least 3 times out of 12 consecutive moves, with no intervening move penetrating that barrier. Thus if a subject approached a position, reversed, made a move or two back, approached the position again, reversed again, and without penetrating it, approached it again and reversed, with no more than 12 total moves occurring from the first to the last approach, it was counted.





from a box (an apparently progressive move). This was the case when the subjects were heading toward the goal, but not when they were heading away from the goal, as is shown in Table 3. The difference between the behavior in the two directions was significant, χ^2 = 9.16 $p < 0.01$. This directional anomaly is somewhat reduced if we consider one of the major characteristics of the problem space of these isomorphs. This is that the first ball is moved on every other move; a fact rapidly recognized by the subjects who therefore quickly learn to not treat adding that ball as a regressive move (nor for that matter, removing it as a necessarily progressive move). The movement of the first ball rapidly becomes almost automatic. If an analysis is performed for all of the remaining moves, the prediction that adding a ball makes a reversal more likely is supported. An analysis of variance shows that there is an effect of adding or subtracting a ball (whether a move appears progressive or regressive, according to our analysis). ($F(1,26) = 5.41$, $p < 0.05$), while there is no effect of direction of movement ($F(1, 26) = 0.29$, $p < 0.75$). There is however an interaction between direction of travel (toward or away from the goal) and the likelihood of a barrier occurring when a ball is added or subtracted ($F(1,26) = 7.93$, $p < 0.01$). When subjects were headed toward the goal, the effect was very strong; almost all of the multiple reversals (barriers) occurred on moves where a ball was added. This was not the case when they were moving away from the goal; they were about equally likely to hit a barrier when adding or subtracting a ball. Given that the subjects were not necessarily aware of moving toward or away from the goal, the reasons for this remaining directional sensitivity are not clear.

Insert Table 3 About Here

A pattern of moves that emerges from examination of Figures 8 and 9, in addition to the tendency of subjects to reverse direction to avoid making a regressive move when headed toward the goal, is their tendency to make very quick progress toward the goal after an oftentimes lengthy period of not making much progress. This latter behavior is particularly striking since it occurs in so many of the move records. What this suggests is some sudden insight or increment in ability that, when it occurs, results in a rapid solution of the problem. The striking feature of this dash to a solution is that it often occurs after hundreds of moves that have yielded no progress at all, with the result that subjects traverse the whole solution path in their last 21 to 30 moves. What is the nature of the insight or learning that enabled this sudden solution to be obtained? In order to investigate

Table 3: The likelihood of reversing direction as a function of position and direction of travel toward or away from the goal. This function is listed separately for all moves and for all but the repetitive first ball move.

	Direction	
	Toward Goal	Away from Goal
All Balls		
Reversals when adding ball	91	53
Reversals when removing ball	56	71
Other Than First Ball		
Reversals when adding ball	76	36
Reversals when removing ball	9	38

that question, a number of learning measures were developed.

Learning and Transfer

We identified three types of useful learning that could conceivably occur in these problems. These kinds of learning could both account for the final sprint to a solution, and provide subjects with a useful source of transfer information to hasten the solution of the second problem. These modes of learning are:

1. Learning not to reverse direction (*i.e.* not take back immediately a move just made),

2. Learning the move restrictions that define legal moves (to avoid illegal moves),

3. Learning to compile or "chunk" small sets of consecutive moves (to make planning easier).

In addition, we defined an overall measure of learning: the number of moves subjects actually took to reach the goal from position x divided by the distance of x from the goal. We examined x=21, 15, 10, and 7. The usefulness of these three measures as indices of learning was tested in two ways. We tested whether the measure did change from the initial problem to the transfer problem, i.e. whether the behavior in question did show improvement from initial to transfer problem.

We also carried out a more precise subject-by-subject comparison within each problem condition. This was accomplished by correlating the amount of learning on the initial problem with the subject's performance on the target problem that was paired with it. This measure is more appropriately designated a measure of transfer, but because it measures the learning achieved in the first problem it serves as a measure of learning as well.

First we will look at reversals, illegal moves, and rate of progress toward the goal. The numbers of reversals made in solving the pair of problems are presented in Table 4. We see that there was a decrease in reversals from the initial problem to the transfer problem for the Lo-Info subjects in the 21-move problems, but not for the No-Info or All-Info subjects. Table 4 also shows that there was no corresponding reduction in the number of illegal moves from first to second problem, except for the All-Info condition.

Despite the absence of a general decrease from initial problem to transfer problem in the number of illegal and reversal moves, comparison of the subjects' behavior at the beginning and end of each problem shows that there was a decrease in the number of illegal and reversal moves over the course of solving both the initial and transfer problems. Table 4 shows that in both positions, for every proolem type, the number of illegal and reversal moves made in the first 20 moves was larger than the number made in the last 20 moves. It is not clear why subjects who have learned to avoid illegal and reversal moves on an initial problem should, in most cases, have to learn to do so again on the second problem. The data presented in Table 4 clearly demonstrate that this is the case.

Insert Table 4 About Here

An overall measure of problem end-game performance, the number of moves it took subjects to reach the goal from their last occupancy of the position 21 moves away (*i.e.* their last visit back to the beginning of the problem) is also shown in Table 4. We see that for the Lo-Info and All-Info conditions, the number of moves to solve the problem from that position is very small, and that for all problem conditions, this number of moves is quite small compared to the total number of moves to solution. The subjects did demonstrate efficacious move-making at the problems' end, and for the Lo-Info problem in particular, the solution was accomplished in close to the minimum number of moves. There was also improvement from first to second problem for at least some of the problem conditions.

This finding contrasts with whole problem measures such as the number of illegal moves, which do not show much improvement from initial to target position for most of the sets of problems. Learning the legality of moves as a determinant of transfer does not seem to play a major role in this experiment even though there are large differences in the proportion of illegal moves made in different problem conditions.

Our final comparison seeks to correlate how much the subjects appeared to understand by the time they completed the first problem with their performance on the transfer problem. Understanding was measured (a) by the number of moves required to solve the initial problem after position 15 was reached; and (b) by the number of moves required to solve the initial problem after position 10 was reached. (c) by the number of reversals in the last 20 moves, and (d) the number of illegal moves in the last 20 moves. The predictive power

Table 4: Reversals: The proportion of moves that return the solver to the immediately preceding state by problem type and position, and Illegals: the proportion of moves that violate one of the problem rules. These measures are also presented for the beginning (first 20 moves) and end (last 20 moves) of the problem. The Last Visit data presents The number of moves from the last occupancy of the problem's starting position (positon 21). The minimum number of moves to solve the problem from this position is 21. In some problem conditions the number of moves was close to that optimal number.

		Problem Type	
	No-Info	Lo-Info	All-Info
Reversals/Total Moves:			
Problem 1	0.14	0.29	0.19
Problem 2	0.13	0.18	0.19
Illegals/Total Moves:			
Problem 1	0.54	0.05	0.29
Problem 2	0.53	0.07	0.17
Moves From Last Occupancy of Position 21:			
Problem 1	91.3	38.2	32.8
Problem 2	56.8	24.7	34.4
Illegals/Moves, Problem 1:			
First 20	0.59	0.19	0.35
Last 20	0.43	0.01	0.08
Illegals/Moves, Problem 2:			
First 20	0.53	0.11	0.30
Last 20	0.40	0.00	0.10
Reversals/Moves, Problem 1:			
First 20	0.34	0.29	0.33
Last 20	0.20	0.18	0.10
Reversals/Moves, Problem 2:			
First 20	0.25	0.18	0.28
Last 20	0.19	0.10	0.13

of these "endgame" measures was compared to the predictive power of an overall measure of subject "ability" on these problems. (e) the time required to solve the first problem within each problem pairing.

If transfer occurred, we would expect negative correlations for measures (a) and (b), (the larger the slope--indicating a faster approach to the goal on the first problem-- the shorter the expected time to solve the second), and a positive correlation, for (c), (d), and (e). The results were in the predicted direction for all but (e) the illegal moves measure which was as likely to show a negative as a positive correlation. For each of the slope measures there was a moderate to large correlation (in the range 0.4 to 0.98) in 7 of the 13 problem-pair categories, and all but one were in the expected direction. For the reversal measure the outcome was similar, but held in 8 of the 13 cases with 1 in the wrong direction. For the time measure, the results were 5 in the expected direction and 2 in the opposite, but the correlations tended to be smaller. The slope and reversal measures were thus generally better predictors of performance on the second problem than was the initial problem solution time.

We conclude that subjects' behaviors in the final stages of solving the initial problem are predictive of their behaviors on the transfer problem. The speed with which they close in on the goal appears to be a valid indication that they have learned something transferable about the problem solution.

Discussion

The experiments reported here investigated two major aspects of problem solving in a particular problem milieu. These are (1), the nature of the search space and its effect on problem solving behavior, and (2), the nature of the move operator and its effect on both problem solving behavior and transfer of skill from one problem to a subsequent one. We will discuss each of these issues separately.

1. Difficulty

The major finding to emerge from this work is that the search space, in the range of problems examined here, is not an important determinant of the difficulty of the problem. We have presented three types of evidence for this conclusion. As is true of other work on isomorphic problems, the current study reveals very large differences in problem difficulty between problems with identical search spaces. The Chinese Ring Puzzle was extremely difficult. It was solved by only one half of the subjects in a two hour time period. even after they were given a strong demonstration of how to make a move; and it was solved by essentially no subjects when no move hint was given. This was the case in both initial and transfer position: having another isomorph precede it did not make it any more solvable. The digital isomorphs, on the other hand, were usually solved in 10 to 20 minutes in initial position, and 3 to 15 minutes in transfer position.

Experiment 4 provided a more direct test of the hypothesis that search space does not determine difficulty by presenting subjects with isomorphs that differed only in search space. The comparison of the 21-move and 31-move problems showed that the length of the search space was not a significant source of difference in problem difficulty.

Finally, we found very little transfer across some of the isomorphs. If the search space was an important determinant of the problem solving behavior, then we would expect a lot of transfer between problems that shared an identical search space. This clearly did not happen. While some of the problems were effective as sources or recipients of transfer, others were not. The important variables were not problem search space variables, but representational features of the problems, such as the nature of moves and the presence or absence of legality cues.

The move operator was a major determinant of problem difficulty in the studies reported here. The analog nature of the Chinese Ring Puzzle, where the moves were, at least initially, not discrete, discriminable, and easily encodable, made that problem extremely difficult. The evidence for this conclusion includes (a) the finding that the move hints (both hint 2 and hint 3) did make the problem solvable for a significant proportion (1/2) of the subjects, and (b) the finding that the same problem with digitized move operators (the No-Info problem) was very easy when compared to the Chinese Puzzle. Insofar as problem difficulty is concerned, the move operator is the preponderant factor. When taken together with similar findings that derive from work in a variety of problem domains (Hayes and Simon, 1977, Kotovsky, Hayes, and Simon, 1985, Carroll, Thomas, and Malhotra, 1980, Wason and Johnson-Laird, 1972, and Wason and Shapiro, 1971) the generality of this finding is supported.

2. Transfer

The transfer of skill learned on one problem to a problem presented subsequently is an issue of theoretical and pedagogical interest. The work reported here presents a number of findings about transfer;

1. When a source problem is easy because a cue provides a "crutch" (where a crutch is defined as a problem solving aid that points to the correct move without providing any understanding or mental model of the problem) the removal of the crutch will result in less transfer than would otherwise be expected. The evidence comes from the Lo-Info problem as initial problem. The Lo-Info problem did not produce more transfer than the No-Info problem, even though it presented information about move legality that was very effective in reducing problem difficulty.

2. The same crutch-like cue that produced little added transfer when the Lo-Info problem was a source problem strongly facilitated transfer <u>to</u> that problem. We conjecture that this was because the Lo-Info problem was easy enough to allow subjects, while solving it, to think about its relation to the problem they had solved previously. A more surprising related finding was that the Chinese Ring Puzzle, even when not solved, tended to produce positive transfer to the Lo-Info problem.

3. There was no transfer to the difficult Chinese Ring Puzzle from the problem that preceded it. This finding is supported by work with isomorphs of the Tower of Hanoi, which has a quite different problem space. (Hayes & Simon, 1974, 1977; Kotovsky, Hayes & Simon, 1986). We propose the same explanatory mechanism as that suggested above. When a transfer problem is very hard, no working memory capacity is left for attending to and using what was learned from an initial problem. It is of course possible, even on very hard problems such as the Chinese Ring Puzzle, to learn to produce rapid solutions, for example, if the problem is administered repeatedly, as was done by Ruger (1910) who had people solve it up to 50 times. With repeated solution attempts, the problem becomes easier through a process that can be thought of as "self transfer". As we have seen with our digital isomorphs, such self transfer can be effective.

4. The major predictor of transfer was not the size or structure of the problem space but the similarity of move operators between the initial problem and the transfer problem. There was a great deal of transfer between the Lo-Info, No-Info, and All-Info problems, but very little transfer between the Chinese Ring Puzzle and any other problem except possibly the Lo-Info condition in transfer position. The transfer results thus parallel the findings about problem difficulty: the determining factor is the nature of the move operator rather than the problem search space.

We have presented data on a set of problem isomorphs of the Chinese Ring Puzzle. It is a problem that should, by traditional measures (size and branchiness of problem search space, amount of knowledge or expertise required), be very easy to solve. It is extremely difficult. We have shown that the difficulty resides largely in the move operator: isomorphs with difficult-to-discover analog move operators are inordinately difficult, while those with moves that are discrete and easily defined are much easier. In these problems, the limited processing resources the subjects initially bring to the problems are consumed by the task of discovering the nature of the move, to the point where they cannot do the planning, placekeeping, or other simple processing that allow a solution to be found. The same types of capacity limitations limit transfer to more difficult isomorphs.

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